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TURBULENT HEAT TRANSFER IN A FLOW OF LIQUID METAL NEAR THE WALL

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The article discusses turbulent heat transfer in media with small Prandtl numbers ($Pr \ll 1$ for liquid metals). In this case, the thermal sublayer is Pr^{-1} times thicker than the viscous sublayer. It is established that the coefficient of turbulent heat transfer varies in the thermal sublayer proportionally to the second power of the distance to the wall; the ratio of the coefficients of the turbulent transfer of heat and momentum in this region decreases in accordance with a linear law with approach to the wall. The conclusions of the theory are compared with the experimental data of other authors.

As is well known, the Prandtl numbers for liquid metals are small: $Pr = \nu/\chi \sim 10^{-2} \ldots 10^{-3}$ (ν is the kinematic viscosity; χ is the coefficient of thermal diffusivity), and, with $\chi \gg \nu$. the region of an influence of the molecular effects of heat transfer ("the thermal sublayer") is far larger than the viscous sublayer, whose dimensions are determined by the scale $y_1 = \nu/v_*$ ($v_* = \sqrt{\tau_w/\rho}$ is the parameter of the "dynamic velocity"; τ_w is the friction stress at the wall; ρ is the density of the liquid). The thickness of the thermal sublayer with $Pr \ll 1$ is determined by the scale $y_2 = \chi/v_*$ [1]. Outside the thermal sublayer, in the layer of constant friction stress, considerations of dimensionalities give a value of $\chi_T(y) = \text{const}\ v_* y$, where $\chi_T(y)$ is the coefficient of turbulent thermal diffusivity; y is the distance to the wall. The behavior of the function $\chi_T(y)$ in the thermal sublayer is determined in accordance with the equation for the pulsations of the temperature.

1. Let us consider the turbulent flow of an incompressible liquid, flowing above a smooth surface in the direction of the x axis; we direct the y axis along a normal to the wall; the z axis is perpendicular to the x and y axes. We denote by U(y) the mean velocity of the flow, and by u, v, w the pulsational components of the velocity in the x, y, z directions, respectively. The turbulence is assumed to be statistically steady-state with respect to the time and homogeneous with respect to the coordinates x and z.

We limit ourselves to a consideration of the region near the wall $y \ll L$ (L is the external scale of the flow), where the turbulence has a universal character [1, 2]. The basic premises of the theory of the similarity of flow near the wall are formulated in the form of two hypotheses, analogous to the Kolmogorskii similarity hypotheses [1]:

1. In the case of turbulence near the wall with sufficiently large Reynolds numbers Re, the statistical conditions of turbulence of the pulsations of the velocity in a region located close to a smooth wall are uniquely determined by two parameters: v_* and ν.

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2. If $Re \gg 1$, there exists a broad region $L \gg y \gg y_1$, in which statistical conditions of the pulsations of the velocity are uniquely determined by the parameter v_* and do not depend on ν.

By statistical conditions there should be understood the ensemble of all the multidimensional distributions of the probability or the set of all the multipoint moments of the pulsational velocities. Hypothesis 2 can turn out to be invalid if the observation points are taken too close together, i.e., at distances on the order of the scale of the smallest eddies ν/v_*. For the hypothesis to be valid, at least one of the conditions must be satisfied

$$|\mathbf{x} - \mathbf{x}'| \gg \frac{\nu}{v_*}, \quad |t - t'| \gg \frac{\nu}{v_*} \tag{1.1}$$

for any given points $\mathbf{x}$, $\mathbf{x}'$ and moments of time t, t' at which are taken the values of the velocities entering into the hydrodynamic moments.

In the region $L \gg y \gg y_1$ let us consider the moment $\langle v(\mathbf{x}, t) v(\mathbf{x}', t)\rangle$ (the angular brackets denote averaging with respect to the time or with respect to the statistical ensemble). Assuming the satisfaction of the first condition of (1.1) and the validity of hypothesis 2, we conclude that it depends only on v_*, y, y', $x'-x$, $z'-z$ (the differences arise by virtue of inhomogeneity with respect to x and z). Since from v_* and the coordinates a dimensionless combination cannot be constructed, the overall form of the moment must be the following:

$$\langle v(\mathbf{x}, t) v(\mathbf{x}', t)\rangle = v_*^2 R\left(\frac{y'}{y}, \frac{x'-x}{y}, \frac{z'-z}{y}\right), \tag{1.2}$$

where $R(\eta, \xi, \zeta)$ is some dimensionless function. Analogously, expressions are established for any arbitrary moments. In what follows, we require the following:

$$\langle u_i(\mathbf{x}, t) u_j(\mathbf{x}', t) u_k(\mathbf{x}'', t)\rangle = v_*^3 \Phi_{ijk}\left(\frac{y'}{y}, \frac{x'-x}{y}, \frac{z'-z}{y}; \frac{y''}{y}, \frac{x''-x}{y}, \frac{z''-z}{y}\right); \tag{1.3}$$

$$\left\langle u_i(\mathbf{x}, t) \frac{\partial u_j(\mathbf{x}', t)}{\partial t}\right\rangle = \frac{v_*^3}{y} \Psi_{ij}\left(\frac{y'}{y}, \frac{x'-x}{y}, \frac{z'-z}{y}\right), \tag{1.4}$$

where $u_i(\mathbf{x}, t)$ is the vector of the pulsational velocity; Φ_{ijk}, Ψ_{ij} are dimensionless functions. In the case where different moments of time are taken in (1.2)-(1.4), in the dimensionless functions, there arises a dependence on the complexes $[(t'-t)/y]v_*$, $[(t''-t)/y]v_*$.

The law of relative change in the mean velocity has the form [1]

$$\Delta U = U(y') - U(y) = \frac{v_*}{\varkappa} \ln \frac{y'}{y}, \tag{1.5}$$

where $\varkappa = 0.4$ is the Karman constant, and y and y' are taken in the region where hypothesis 2 is valid.

Formulas of the type of (1.2)-(1.4) were discussed by Townsend [2]. The theory of turbulence near the wall is set forth in [1, 2].

2. The equations for the mean temperature $T(y)$ and the pulsation of the temperature $\theta(\mathbf{x}, t)$ are obtained from the equation of convective heat transfer under the assumption of a statistical steady state with respect to the time and of homogeneity with respect to the coordinates x and z,

$$\frac{d\langle v\theta\rangle}{dy} = \chi \frac{d^2T}{dy^2}; \tag{2.1}$$

$$\Delta\theta = \frac{v}{\chi}\frac{dT}{dy} + \frac{1}{\chi}\left\{U\frac{\partial\theta}{\partial x} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} - \frac{d\langle v\theta\rangle}{dy} + \frac{\partial\theta}{\partial t}\right\}, \tag{2.2}$$

where Δ is a Laplace operator. The boundary conditions to Eqs. (2.1), (2.2) can be of two types:

$$(T)_{y=0} = \text{const}, \quad (\theta)_{y=0} = 0; \tag{2.3}$$

$$\left(\frac{dT}{dy}\right)_{y=0} = \text{const}, \quad \left(\frac{\partial\theta}{\partial y}\right)_{y=0} = 0. \tag{2.4}$$

The temperature pulsations in the thermal sublayer are brought about by the turbulent field of the velocity, imposed on the linear profile of the mean temperature ($dT/dy \cong$ const in the thermal sublayer). It is natural to postulate that, with $y \ll y_2 = \chi/v_*$, the space-time scales of the temperature pulsations are determined only by the scales of the turbulent motions and do not depend on the parameter χ (here χ affects only the amplitude θ). If $y \gg y_1 = \nu/v_*$, then in accordance with the results of Sec. 1, at a distance y from the wall the scale of the eddies $\sim y$, and the time scale of the motion $\sim y/v_*$. Evaluating the derivatives

$\theta(\mathbf{x}, t)$ by the scales y and y/v_*, and the velocity by the parameter v_*, we conclude that, in a thermal sublayer (with $y \gg y_2$), the following inequality holds:

$$\frac{1}{\chi}\left\{U\frac{\partial\theta}{\partial x}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}-\frac{d\langle v\theta\rangle}{dy}+\frac{\partial\theta}{\partial t}\right\}\sim\frac{v_*\theta}{\chi y}\ll\frac{\theta}{y^2}\sim\Delta\theta,$$

making it possible to neglect the terms in shaped brackets in (2.2) in comparison with the term $\Delta\theta$ (the evaluations have meaning, of course, not for individual implementations, but only for mean-square values of the quantities).

For a convincing demonstration of the possibility of neglecting the above-mentioned terms in (2.2), we let χ and dT/dy approach infinity in such a way that their ratio will remain finite (this condition is fulfilled, since dT/dy can vary independently of χ). After this, in the right-hand part of (2.2) there remains only $(v/\chi)(dT/dy)$, and the remaining terms drop out, since they contain the factor χ^{-1}. By virtue of the linearity of Eq. (2.2) with respect to θ, dT/dy determines the absolute value of θ, but has no effect on the relationship between the terms of this equation itself.

Thus, it is certain that, with very large, but finite, values of χ ($\chi \gg \nu$), the temperature pulsations in the thermal sublayer can be described in the first approximation by the Poisson equation (analogous to the approach used in [3] with determination of the spectrum of the temperature pulsations in the case of isotropic turbulence with $Pr \ll 1$). The solution of the approximate Eq. (2.2) has the form

$$\theta_1(\mathbf{x}, t) = -\frac{1}{\chi}\iiint\limits_{y'\geq 0} d^3\mathbf{x}' G(\mathbf{x}, \mathbf{x}')\, v(\mathbf{x}', t)\frac{dT(y')}{dy'}, \qquad (2.5)$$

where $G(\mathbf{x}, \mathbf{x}')$ is a Green function of the Poisson equation for the region $y \geq 0$;

$$G(\mathbf{x}, \mathbf{x}') = \frac{1}{4\pi}\left[\frac{1}{\sqrt{(y'-y)^2+(x'-x)^2+(z'-z)^2}} \mp \frac{1}{\sqrt{(y'+y)^2+(x'-x)^2+(z'-z)^2}}\right]. \qquad (2.6)$$

In (2.6), the minus sign relates to the condition (2.3), and the plus sign to the condition (2.4). Multiplying (2.6) by $v(\mathbf{x}, t)$ and averaging, we obtain an expression for the turbulent heat flux:

$$\langle v\theta_1\rangle = -\frac{1}{\chi}\iiint\limits_{y'\geq 0} d^3\mathbf{x}' G(\mathbf{x}, \mathbf{x}')\langle v(\mathbf{x}, t)\, v(\mathbf{x}', t)\rangle\frac{dT}{dy}. \qquad (2.7)$$

It is postulated, in accordance with (1.2), that the correlation function decreases rapidly with $|\mathbf{x}'-\mathbf{x}| > y$, so that the region of integration making the principal contribution to the integral (2.7) has the dimension $\sim y$. Therefore, the gradient dT/dy can be taken out from under the integral sign, since $dT/dy \cong \text{const}$ with $y \ll y_2$.

We go over in (2.7) to the new variables

$$\eta = \frac{y'}{y};\quad \xi = \frac{x'-x}{y};\quad \zeta = \frac{z'-z}{y}, \qquad (2.8)$$

on which depend the moments $\langle v(\mathbf{x}, t)v(\mathbf{x}', t)\rangle$, in accordance with formula (1.2). Here y^3 is dropped out of the expression for $d^3\mathbf{x}'$, and y^{-1} out of the Green's function (2.6), and it assumes the form

$$\bar{G}(\eta, \xi, \zeta) = \frac{1}{4\pi}\left[\frac{1}{\sqrt{(1-\eta)^2+\xi^2+\zeta^2}} \mp \frac{1}{\sqrt{(1+\eta)^2+\xi^2+\zeta^2}}\right].$$

Dividing expression (2.7) by dT/dy and changing sign, in accordance with the definition of the coefficient of turbulent thermal diffusivity, we obtain

$$\chi_T^{(1)}(y) = \frac{v_*^2 y^2}{\chi}\iiint\limits_{\eta\geq 0} d\eta\, d\xi\, d\zeta\, \bar{G}(\eta, \xi, \zeta)\, R(\eta, \xi, \zeta) = C\chi y_-^2, \qquad (2.9)$$

where $y_- = v_* y/\chi$; C is a universal constant, characterizing the integral; its calculation is impossible, since the function $R(\eta, \xi, \zeta)$ from (1.2) is unknown. The Green's function and $R(\eta, \xi, \zeta)$ have positive maxima with $\eta = 1$, $\xi = \zeta = 0$. Since, with large values of η, ξ, ζ, these functions fall rapidly, it can be assumed that $\chi_T^{(1)}(y) > 0$. Formula (2.9) gives the first term of an expansion in small values of y_-.

The terms discarded in the first approximation can be taken into consideration in accordance with the theory of perturbations. Substituting into them the value of $\theta_1(\mathbf{x}, t)$ from (2.5), we find $\theta_2(\mathbf{x}, t)$, i.e., a solution in the second approximation. After multiplication by $v(\mathbf{x}, t)$ and averaging, we obtain

$$\langle v\theta_2\rangle = \frac{1}{\chi^2}\frac{dT}{dy}\iiint\limits_{y'\geq 0} d^3\mathbf{x}'\iiint\limits_{y''\geq 0} d^3\mathbf{x}'' G(\mathbf{x}, \mathbf{x}')\left[\left\langle v\frac{\partial v''}{\partial t}\right\rangle + \Delta U\,\langle vv''\rangle\frac{\partial}{\partial x'} + \langle vu_j'v''\rangle\frac{\partial}{\partial x_j'}\right]G(\mathbf{x}', \mathbf{x}''), \qquad (2.10)$$

where the primes with the functions denote dependence on the coordinates x' or x"; j indicates summation, $u'_j = (u'v', w')$, $x'_j = (x', y', z')$. In (2.10), the system of coordinates x' is assumed to be moving at the velocity of the liquid at a distance y from the wall. In accordance with (1.5) $\Delta U = (v_*/\varkappa)\ln(y'/y)$ is the mean velocity of the flow in this system of coordinates. In accordance with the Galileo principle of relativity, a transition to a moving system of reckoning cannot change the value of the flows of heat and momentum in the direction of the y axis.

We go over in (2.10) to the variables (2.8) and to an analogous set of three variables with x". Under these circumstances, d^3x' and d^3x'' give y^6 and two green's functions $- y^{-2}$, while the terms in square brackets, in accordance with (1.2)-(1.5), give the factor v^3_*/y. In accordance with the definition of the quantity $\chi_T(y)$, from (2.10) we obtain the correction to (2.9): $\chi_T^{(2)}(y) = \text{const}\ \chi y_-^3$, which confirms once again the validity of the evaluations made [since $\chi_T^{(2)} \ll \chi_T^{(1)}$ with $y_- \ll 1$].

It remains to establish the value of $\chi_T(y)$ in the depths of the viscous sublayer with $y \ll y_1$. Here, in accordance with hypothesis 1, all the quantities in (2.7) are rendered dimensionless with the use of v_* and ν. As a result of the equation of continuity, v(x, t) with $y_+ = v_* y/\nu \ll 1$ varies according to the law $v \sim v_* y_+^2$ [1]. Taking into account that G(x, x') behaves in a different manner, we obtain from (2.7) $\chi_T(y) \sim \nu \Pr y_+^3$ for conditions (2.3), and $\chi_T(y) \sim \nu \Pr y_+^2$ for conditions (2.4).

Outside of the viscous sublayer, the formula $\nu_T = \varkappa v_* y$ holds for the turbulent viscosity. Dividing the value of χ_T from (2.9) by ν_T, we obtain the relationship $\gamma = \Pr_T^{-1} = (C/\varkappa)y_-$ ($\Pr_T$ is the turbulent Prandtl number). In accordance with the existing experimental data, γ decreases with an approach to the wall. In Fig. 1 the points illustrate the data of different authors, collected in [4]. Unfortunately, data are lacking for the region $y_- \ll 1$, where the proposed theory is formally valid.

For the value of χ_T, the following interpolation formula is proposed:

$$\frac{\chi}{\chi_T} = \frac{A}{y_-} + \frac{B}{y_-^2} \quad (A, B - \text{const}), \tag{2.11}$$

giving a correct dependence with large and small values of y_- [in (2.11) the change in the behavior of χ_T in the viscous sublayer is neglected]. We substitute χ_T from (2.11) into Eq. (2.1) and integrate it taking account of the boundary condition $\chi(dT/dy)_{y=0} = q_w/\rho c_p$ (q_w is the density of the flow at the wall; c_p is the specific heat capacity of the liquid with constant pressure). Setting A = 2.2, B = 6.5, we obtain the profile of the mean temperature:

$$\frac{\rho c_p v_*}{q_w} T = 2.54 \lg(1 + 0.338 y_- + 0.154 y_-^2) + 1{,}78 \operatorname{arctg}\left(\frac{y_-}{0.478 y_- + 2.83}\right), \tag{2.12}$$

in good agreement with the experimental data of [5]. In Fig. 2, the vertical segments illustrate the scatter of these data; the curve is drawn in accordance with formula (2.12).

Using (2.11), for the ratio γ we obtain the expression

$$\gamma = \frac{y_-}{\varkappa(Ay_- + B)} = \frac{y_-}{0.88 y_- + 2.6},$$

in accordance with which the curve in Fig. 1 was drawn. In [4], an analogous expression was obtained on the basis of averaged equations and the hypothesis of closure.

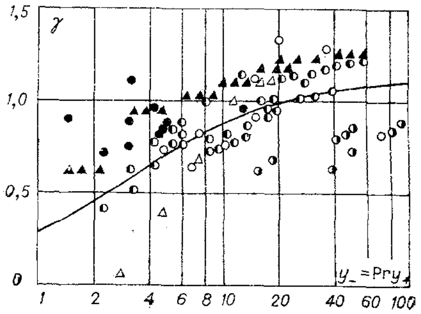

Fig. 1

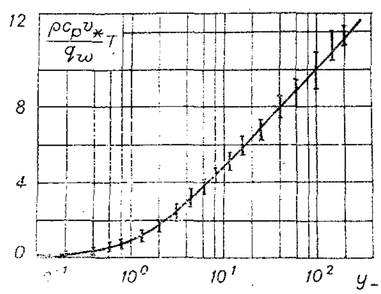

Fig. 2

Thus, in the case $Pr \ll 1$, the value of Pr_T, in the greater part of the thermal sublayer, varies proportionally to the distance to the wall. In this case there is no analogy between the turbulent transfers of heat and momentum.

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WAVE FORMATION WITH THE COMBUSTION OF CONDENSED SUBSTANCES IN A TURBULENT FLOW

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The surface of samples of powder burning in a flow in the presence of erosion is spotted with roughnesses of almost-periodic structure [1]. Such roughnesses are also observed with the combustion of some ablating materials in a flow [2]. It has been observed that, with the unstable (resonance) combustion of powders, different acoustical modes correspond to the structure of the roughnesses [3]. One of the possible mechanisms of the formation of waves on the surface, developed in [4, 5], has still not received sufficient experimental confirmation. The present article discusses the laws governing the formation of a wave structure on the surface of various condensed substances, burning in a turbulent flow of the combustion products of ballistic powder N.

The experiments were made in a unit, analogous to that described in [6], and consisting of a gas generator with an erosion nozzle, a device for letting down the pressure, and a counterpressure block. A charge of ballistic powder N was put into the combustion chamber. The erosion nozzle ensured the possibility of blowing the sample under investigation with powder gases. The velocity of the gas flow and the level of the pressure were regulated by a change in the parameters of the gas generator. Extinction was effected by letting down the pressure with the sudden opening of an opening on the side of the combustion chamber. The starting parameters (the combustion surface, the critical cross section, etc.) were so selected as to exclude the appearance of instability or resonance combustion. Thus, in all the experiments, the combustion took place under steady-state conditions.

The investigations were made on samples made of Capron, vinyl plastic, ebonite, Plexiglas, fluorine plastic, polyethylene, textolite, and graphite, with a constant pressure of $75 \cdot 10^5$ N/m^2 and velocities of the blowing of 10-600 m/sec. The samples were cylindrical, with a diameter of $1.7 \cdot 10^{-2}$ m and a length of 0.1 m.

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